PERIODIC ORBITS OF THE SITNIKOV PROBLEM VIA A POINCARÉ MAP

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Abstract. In this paper by means of a Poincaré map, we prove the existence of symmetric periodic orbits of the elliptic Sitnikov problem. Furthermore, using the presence of the Bernoulli shift as a subsystem of that Poincaré map, we prove that not all the periodic orbits of the Sitnikov problem are symmetric periodic orbits.

Key words: Sitnikov motions, periodic orbits, Poincaré map, symmetric periodic orbits

1. Introduction

We consider the elliptic Sitnikov problem, here called simply the *Sitnikov problem*. It is a special case of the restricted three-body problem where the two primaries with equal masses are moving in an elliptic orbit of the two-body problem, and the infinitesimal mass is moving on the straight line orthogonal to the plane of motion of the primaries which passes through the center of mass. The purpose of this paper is to use the symmetries of the problem to find symmetric periodic solutions of the Sitnikov problem. We do this by means of a Poincaré map, which is essentially the Poincaré map that was used by Alekseev (1968) and Moser (1973) to analyze the final evolutions of the orbits of the Sitnikov problem. The presence of the Bernoulli shift as a subsystem of that Poincaré map will allow us to prove the existence of non-symmetric periodic orbits.

In Section 2 we give the equations of motion of the Sitnikov problem and we prove that the flow of the Sitnikov problem is complete (that is, all solutions are defined for all $t \in \mathbb{R}$), see Proposition 2.

In Section 3 we define the Poincaré map f_e and we analyze the properties of f_e^n for $n = 1, 2, \dots$.

In Section 4 we analyze the symmetries of the problem and we see how they can be used to obtain periodic orbits, see Propositions 5, 6 and 7. Moreover, we characterize the symmetric periodic orbits and, by means of the Poincaré map, we prove the existence of symmetric periodic orbits of the Sitnikov problem for all values of the eccentricity $e \in (0, 1)$, see Propositions 12 and 15.

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