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FAMILIES OF PERIODIC ORBITS FOR THE SPATIAL ISOSCELES 3-BODY PROBLEM*

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Abstract. We study the families of periodic orbits of the spatial isosceles 3-body problem (for small enough values of the mass lying on the symmetry axis) coming via the analytic continuation method from periodic orbits of the circular Sitnikov problem. Using the first integral of the angular momentum, we reduce the dimension of the phase space of the problem by two units. Since periodic orbits of the reduced isosceles problem generate invariant two-dimensional tori of the nonreduced problem, the analytic continuation of periodic orbits of the (reduced) circular Sitnikov problem at this level becomes the continuation of invariant two-dimensional tori from the circular Sitnikov problem to the nonreduced isosceles problem, each one filled with periodic or quasi-periodic orbits. These tori are not KAM tori but just isotropic, since we are dealing with a three-degrees-of-freedom system. The continuation of periodic orbits is done in two different ways, the first going directly from the reduced circular Sitnikov problem to the reduced isosceles problem, and the second one using two steps: first we continue the periodic orbits from the reduced circular Sitnikov problem to the reduced elliptic Sitnikov problem, and then we continue those periodic orbits of the reduced elliptic Sitnikov problem to the reduced isosceles problem. The continuation in one or two steps produces different results. This work is merely analytic and uses the variational equations in order to apply Poincaré's continuation method.

Key words. periodic orbits, quasi-periodic orbits, 3-body problem, analytic continuation method

AMS subject classifications. 70F15, 37N05

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1. Introduction.

We consider a special case of the spatial 3-body problem, the *spatial isosceles* 3*-body problem*, or simply the *isosceles problem*. This problem consists of describing the motion of two equally massive bodies, $m_1 = m_2 = 1/2$, having initial conditions and velocities symmetric with respect to a straight line which passes through their center of mass, and a third body, with mass $m_3 = \mu$, having initial position and velocity on this straight line. This problem is called the isosceles problem because the three bodies form an isosceles triangle at any time, eventually degenerated to a segment.

The most interesting application of the spatial isosceles 3-body problem was given by Xia in [25]. He used two spatial isosceles 3-body problems to prove that five bodies can escape to infinity in a finite time without collision. Other works on the spatial isosceles 3-body problem are [16] and the references therein. If in the spatial isosceles 3-body problem the initial positions and velocities of the three bodies are contained in a plane, then the motion remains always in this plane, and we have the so-called *planar isosceles* 3*-body problem*. There are several papers about the planar isosceles 3-body problem, for instance, [9], [17], etc.

When the third body of the isosceles 3-body problem has infinitesimal mass (i.e., $\mu = 0$) then we obtain the *restricted isosceles problems*. Depending on the motion

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