THE GLOBAL FLOW OF THE PARABOLIC RESTRICTED THREE-BODY PROBLEM

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Abstract. We have two mass points of equal masses $m_1 = m_2 > 0$ moving under Newton's law of attraction in a non-collision parabolic orbit while their center of mass is at rest. We consider a third mass point, of mass $m_3 = 0$, moving on the straight line L perpendicular to the plane of motion of the first two mass points and passing through their center of mass. Since $m_3 = 0$, the motion of m_1 and m_2 is not affected by the third and from the symmetry of the motion it is clear that m_3 will remain on the line L. The parabolic restricted three-body problem describes the motion of m_3. Our main result is the characterization of the global flow of this problem.

Key words: global flow, restricted three-body problem, Sitnikov problem

1. Introduction and Statement of the Main Result

The three-body problem is one of the fundamental problems in Celestial Mechanics. The general three-dimensional three-body problem has been studied extensively using numerical simulations and by analytical methods. However, this problem is still too complicated for describing its global flow. This has led to study various simplifications or restrictions of it; these include:

1. The restricted three-body problem: one of the masses is assumed to be zero. The motion of the two massive bodies is integrable, but the motion of the third body, under the gravitational action of the other two, is in general not integrable.
2. The planar three-body problem: motion is restricted to a plane, masses are free.
3. The rectilinear three-body problem: motion is restricted to a line, masses are free.

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