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SYMMETRIC PERIODIC ORBITS NEAR HETEROCLINIC LOOPS AT INFINITY FOR A CLASS OF POLYNOMIAL VECTOR FIELDS

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For polynomial vector fields in $\mathbb{R}^3$, in general, it is very difficult to detect the existence of an open set of periodic orbits in their phase portraits. Here, we characterize a class of polynomial vector fields of arbitrary even degree having an open set of periodic orbits. The main two tools for proving this result are, first, the existence in the phase portrait of a symmetry with respect to a plane and, second, the existence of two symmetric heteroclinic loops.

Keywords: Polynomial vector fields; symmetric periodic orbits; heteroclinic loop.

1. Introduction and Main Result

In this paper, we will study the periodic orbits near infinity of a class of polynomial vector fields in $\mathbb{R}^3$. In order to study the behavior of a polynomial vector field near infinity we will use the Poincaré compactification (see Sec. 2 for details). This technique allows us to extend the vector field in $\mathbb{R}^3$ to a unique analytic vector field on the *Poincaré sphere* $\mathbb{S}^3$ and on the *Poincaré ball* $\mathbb{D}^3$, whose boundary, the sphere $\mathbb{S}^2$, plays the role of the infinity for the initial polynomial vector field.

Let P, Q and R be polynomials in the variables x, y and z. We consider the polynomial vector field $X = (P, Q, R)$ in $\mathbb{R}^3$ that satisfies the following conditions

(C_1) The flow of X is invariant under the symmetry $(x, y, z, t) \to (-x, y, z, -t)$. So the phase portrait of X is symmetric with respect to the plane $x = 0$.

(C_2) The maximum of the degrees of P, Q and R is called the *degree of* X. Here we assume that this degree is even and equal to n.

(C_3) The straight line $y = z = 0$ is invariant by the flow of X, it does not contain any singular point and the flow on it goes in the increasing direction of the x-axis.

(C_4) The straight line $y = z = 0$ intersects the boundary of the Poincaré ball at two singular points a and b, the origins of the local charts U_1 and V_1 respectively. Moreover, a is hyperbolic with local unstable manifold $z_2 = z_3 = 0$ in the local chart U_1. Recall that a singular point is *hyperbolic* if the real part of all its eigenvalues is different from zero. Note that by the symmetry of the problem the singular point b is also hyperbolic.

(C_5) The straight line $z_2 = z_3 = 0$ of the local chart U_2 of the Poincaré ball is invariant by the flow of X, it does not contain any singular point