JOURNAL OF MATHEMATICAL PHYSICS **47**, 122701 (2006)

Infinitely many periodic orbits for the rhomboidal five-body problem

Montserrat Corbera[a)]
Department of Digital and Information Technologies, University of Vic, Laura 13, 08500 Vic, Barcelona, Spain

Jaume Llibre[b)]
Department of Mathematics, Autonomous, University of Barcelona, 08193 Bellaterra, Barcelona, Spain

(Received 20 July 2006; accepted 5 October 2006; published online 8 December 2006)

We prove the existence of infinitely many symmetric periodic orbits for a regularized rhomboidal five-body problem with four small masses placed at the vertices of a rhombus centered in the fifth mass. The main tool for proving the existence of such periodic orbits is the analytic continuation method of Poincaré together with the symmetries of the problem.
[DOI: 10.1063/1.2378617]

I. INTRODUCTION

In this paper we consider a particular case of the planar five-body problem defined as follows. We consider a mass $m_0=1$ at the origin of coordinates with zero initial velocity, two small masses $m_1=m_2=\mu\nu_1$ with initial positions and velocities on the x axis symmetric with respect to the origin, and two small masses $m_3=m_4=\mu\nu_2$ with initial positions and velocities on the y axis also symmetric with respect to the origin (see Fig. 1). Our five-body problem consists of describing the motion of the five masses under their mutual Newtonian gravitational attraction. Due to the symmetry of the initial conditions and velocities, the four small bodies form a rhombus with center at m_0 at any time and the mass m_0 remains at rest at the origin. The description of the motion of this five-body problem is called the *rhomboidal five-body problem*.

Although this is a five-body problem it can be formulated as a Hamiltonian system of two degrees of freedom, one is the distance $x \geqslant 0$ of m_1 to the origin and the other is the distance $y \geqslant 0$ of m_3 to the origin (the distances of m_2 and m_4 to the origin are obtained by symmetry). The system has three singularities, the triple collision between m_0, m_1, and m_2, the triple collision between m_0, m_3, and m_4, and the total collision of the five bodies. Due to the symmetries doing a double Levi-Civita transformation we regularize both triple collisions.

When $\mu=0$ the problem is reduced to two collision two-body problems, the collision two-body problem with m_0 and m_1 and the collision two-body problem with m_0 and m_3. Note that if we take into account the five bodies, then really for $\mu=0$ we have instead of the binary collisions m_0 with m_1, and m_0 with m_3, the triple collisions m_0, m_1, and m_2, and m_0, m_3, and m_4. Since the solutions of the collision two-body problem are known we can compute the periodic solutions of the regularized system for $\mu=0$ in a fixed energy level $h<0$. The objective of this paper is to prove that the symmetric periodic orbits of the regularized rhomboidal five-body problem for $\mu=0$ can be continued to symmetric periodic orbits of the regularized rhomboidal five-body problem for $\mu>0$ sufficiently small. The main tool for proving this result is the classical analytic continuation method of Poincaré.

[a)]Electronic mail: montserrat.corbera@uvic.cat
[b)]Electronic mail: jllibre@mat.uab.cat

