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Qualitative Theory of Dynamical Systems

Infinitely Many Periodic Orbits for the Octahedral 7-Body Problem

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Abstract. We prove the existence of infinitely many symmetric periodic orbits for a regularized octahedral 7-body problem with six small masses placed at the vertices of an octahedron centered in the seventh mass. The main tools for proving the existence of such periodic orbits is the analytic continuation method together with the symmetry of the problem.

Keywords. Continuation method, symmetric periodic orbits, 7-body problem.

1. Introduction

In this paper we consider a particular case of the spatial 7-body problem defined as follows. We consider a mass $m_0 = 1$ located at the origin of coordinates with zero initial velocity, two small masses $m_1 = m_2 = \mu\nu_1$ with initial positions and velocities on the x-axis symmetric with respect to the origin, two small masses $m_3 = m_4 = \mu\nu_2$ with initial positions and velocities on the y-axis symmetric with respect to the origin, and finally two small masses $m_5 = m_6 = \mu\nu_3$ with initial positions and velocities on the z-axis symmetric with respect to the origin (see Figure 1). Our 7-body problem consists of describing the motion of the seven masses under their mutual Newtonian gravitational attraction. Due to the symmetry of the initial conditions and velocities, the six small bodies are located at any time in the vertices of an octahedron with center at m_0, and the mass m_0 remains in rest at the origin. We call the *octahedral* 7*-body problem* the study of the motion of this 7-body problem.

Although this is a 7-body problem it can be formulated as a Hamiltonian system of three degrees of freedom, one is the distance $x \geq 0$ of m_1 to the origin, the other is the distance $y \geq 0$ of m_3 to the origin, and the third is the distance $z \geq 0$ of m_5 to the origin (the distances of m_2, m_4 and m_6 to the origin are obtained by symmetry). The system has seven singularities, the triple collisions between m_0, m_1 and m_2, between m_0, m_3 and m_4, and between m_0, m_5 and m_6;