Journal of Geometry and Physics 59 (2009) 321–339

Contents lists available at ScienceDirect

Journal of Geometry and Physics

journal homepage: www.elsevier.com/locate/jgp

Central configurations of three nested regular polyhedra for the spatial 3n-body problem

M. Corbera [a,*], J. Llibre [b]

[a] Departament de Tecnologies Digitals i de la Informació, Universitat de Vic, 08500 Vic, Barcelona, Spain

[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

ARTICLE INFO

Article history:
Received 4 September 2008
Accepted 15 November 2008
Available online 25 November 2008

MSC:
70F10
70F15

Keywords:
3n-body problem
Spatial central configurations
Nested regular polyhedra

ABSTRACT

Three regular polyhedra are called nested if they have the same number of vertices n, the same center and the positions of the vertices of the inner polyhedron $\mathbf{r}_i$, the ones of the medium polyhedron $\mathbf{R}_i$ and the ones of the outer polyhedron $\mathcal{R}_i$ satisfy the relation $\mathbf{R}_i = \rho\,\mathbf{r}_i$ and $\mathcal{R}_i = R\,\mathbf{r}_i$ for some scale factors $R > \rho > 1$ and for all $i = 1, \ldots, n$. We consider 3n masses located at the vertices of three nested regular polyhedra. We assume that the masses of the inner polyhedron are equal to m_1, the masses of the medium one are equal to m_2, and the masses of the outer one are equal to m_3. We prove that if the ratios of the masses m_2/m_1 and m_3/m_1 and the scale factors ρ and R satisfy two convenient relations, then this configuration is central for the 3n-body problem. Moreover there is some numerical evidence that, first, fixed two values of the ratios m_2/m_1 and m_3/m_1, the 3n-body problem has a unique central configuration of this type; and second that the number of nested regular polyhedra with the same number of vertices forming a central configuration for convenient masses and sizes is arbitrary.

1. Introduction

The equations of motion of the N-body problem in the ℓ-dimensional space with $\ell = 2, 3$ are

$$m_i\,\ddot{\mathbf{q}}_i = -\sum_{j=1,\, j\neq i}^{N} G\, m_i\, m_j\, \frac{\mathbf{q}_i - \mathbf{q}_j}{|\mathbf{q}_i - \mathbf{q}_j|^3}, \quad i = 1, \ldots, N,$$

where $\mathbf{q}_i \in \mathbb{R}^\ell$ is the position vector of the punctual mass m_i in an inertial coordinate system and G is the gravitational constant which can be taken equal to one by choosing conveniently the unit of time. We take the center of mass $\sum_{i=1}^{N} m_i\,\mathbf{q}_i / \sum_{i=1}^{N} m_i$ of the system at the origin of $\mathbb{R}^{\ell N}$. The *configuration space* of the N-body problem in $\mathbb{R}^\ell$ is defined by

$$\mathcal{E} = \left\{ (\mathbf{q}_1, \ldots, \mathbf{q}_N) \in \mathbb{R}^{\ell N} : \sum_{i=1}^{N} m_i\,\mathbf{q}_i = 0,\ \mathbf{q}_i \neq \mathbf{q}_j,\ \text{for } i \neq j \right\}.$$

Given a set of masses $m_1, \ldots, m_N$, a configuration $(\mathbf{q}_1, \ldots, \mathbf{q}_N) \in \mathcal{E}$ is *central* if there exists a positive constant λ such that

* Corresponding author. Tel.: +34 938815519; fax: +34 938856900.
E-mail addresses: montserrat.corbera@uvic.cat (M. Corbera), jllibre@mat.uab.cat (J. Llibre).

0393-0440/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.geomphys.2008.11.012