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Central configurations of nested rotated regular tetrahedra

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ABSTRACT

In this paper we prove that there are only two different classes of central configurations with convenient masses located at the vertices of two nested regular tetrahedra: either when one of the tetrahedra is a homothecy of the other one, or when one of the tetrahedra is a homothecy followed by a rotation of Euler angles $\alpha = \gamma = 0$ and $\beta = \pi$ of the other one.

We also analyze the central configurations with convenient masses located at the vertices of three nested regular tetrahedra when one them is a homothecy of the other one, and the third one is a homothecy followed by a rotation of Euler angles $\alpha = \gamma = 0$ and $\beta = \pi$ of the other two.

In all of these cases we have assumed that the masses on each tetrahedron are equal but masses on different tetrahedra could be different.

1. Introduction

The equations of motion of the N-body problem in the three-dimensional Euclidean space are

$$m_i\,\ddot{\mathbf{q}}_i = -\sum_{j=1, j\neq i}^{N} G\, m_i\, m_j \frac{\mathbf{q}_i - \mathbf{q}_j}{|\mathbf{q}_i - \mathbf{q}_j|^3}, \quad i = 1, \dots, N,$$

where $\mathbf{q}_i \in \mathbb{R}^3$ is the position vector of the point mass m_i in an inertial coordinate system and G is the gravitational constant which can be taken equal to 1 by choosing the unit of time conveniently. We fix the center of mass $\sum_{i=1}^{N} m_i\,\mathbf{q}_i / \sum_{i=1}^{N} m_i$ of the system at the origin of $\mathbb{R}^{3N}$. The *configuration space* of the N-body problem in $\mathbb{R}^3$ is

$$\mathcal{E} = \{(\mathbf{q}_1, \dots, \mathbf{q}_N) \in \mathbb{R}^{3N} : \sum_{i=1}^{N} m_i\,\mathbf{q}_i = 0, \mathbf{q}_i \neq \mathbf{q}_j, \text{ for } i \neq j\}.$$

Given positive masses $m_1, \dots, m_N$ a configuration $(\mathbf{q}_1, \dots, \mathbf{q}_N) \in \mathcal{E}$ is *central* if there exists a positive constant λ such that

$$\ddot{\mathbf{q}}_i = -\lambda\,\mathbf{q}_i, \quad i = 1, \dots, N, \tag{1}$$

that is, if the acceleration $\ddot{\mathbf{q}}_i$ of each point mass m_i is proportional to its position $\mathbf{q}_i$ relative to the center of mass of the system and is directed towards the center of mass. In a central configuration system (1) can be written as

$$\sum_{j=1, j\neq i}^{N} m_j \frac{\mathbf{q}_i - \mathbf{q}_j}{|\mathbf{q}_i - \mathbf{q}_j|^{3/2}} = \lambda\,\mathbf{q}_i, \quad i = 1, \dots, N. \tag{2}$$

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