

Survey on central configurations related with regular polyhedra

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Abstract

This paper is a survey on the known results about central configurations related with regular polyhedra.

Key words and expressions: n -body problem, spatial central configurations, regular polyhedra.

AMS (MOS) Subject Classification: 70F10, 70F15.

1 Introduction

Let $\mathbf{q}_1, \dots, \mathbf{q}_N \in \mathbb{R}^\ell$ denote the position of the N punctual masses $m_1, \dots, m_N$ respectively. The motion of these masses is governed by the equations

$$m_i \ddot{\mathbf{q}}_i = - \sum_{j=1, j \neq i}^N G m_i m_j \frac{\mathbf{q}_i - \mathbf{q}_j}{|\mathbf{q}_i - \mathbf{q}_j|^3}, \quad i = 1, \dots, N,$$

where G is the gravitational constant which can be taken equal to one by choosing conveniently the unit of time. The *configuration space* of the N -body problem in $\mathbb{R}^\ell$ with the center of mass at the origin is

$$\mathcal{E} = \{(\mathbf{q}_1, \dots, \mathbf{q}_N) \in \mathbb{R}^{\ell N} : \sum_{i=1}^N m_i \mathbf{q}_i = 0, \mathbf{q}_i \neq \mathbf{q}_j, \text{ for } i \neq j\}.$$

Given positive masses $m_1, \dots, m_N$ a configuration $(\mathbf{q}_1, \dots, \mathbf{q}_N) \in \mathcal{E}$ is *central* if there exists a positive constant λ such that

$$\ddot{\mathbf{q}}_i = -\lambda \mathbf{q}_i, \quad i = 1, \dots, N.$$