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ORIGINAL ARTICLE

# On the existence of central configurations of $p$ nested regular polyhedra

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**Abstract** In this paper we prove, for all $p \geqslant 2$, the existence of central configurations of the $pn$-body problem where the masses are located at the vertices of $p$ nested regular polyhedra having the same number of vertices $n$ and a common center. In such configurations all the masses on the same polyhedron are equal, but masses on different polyhedra could be different.

**Keywords** $N$-body problems · Spatial central configurations · Nested regular polyhedra

**Mathematics Subject Classification (2000)** 70F10 · 70F15

## 1 Introduction

We consider the $N$-body problem

$$m_k\,\ddot{\mathbf{q}}_k = -\sum_{j=1,\ j\neq k}^{N} G\,m_k\,m_j\,\frac{\mathbf{q}_k-\mathbf{q}_j}{|\mathbf{q}_k-\mathbf{q}_j|^3}\,,\quad k=1,\ldots,N,$$

where $\mathbf{q}_k \in \mathbb{R}^\ell$, with $\ell = 2, 3$, is the position vector of the punctual mass $m_k$ in an inertial coordinate system, and $G$ is the gravitational constant which can be taken equal to one by choosing conveniently the unit of time. By fixing the center of mass $\sum_{k=1}^{N} m_k\,\mathbf{q}_k / \sum_{k=1}^{N} m_k$ of the system at the origin of $\mathbb{R}^{\ell N}$, the *configuration space* of the spatial $N$-body problem is

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