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ON THE EXISTENCE OF BI–PYRAMIDAL CENTRAL CONFIGURATIONS OF THE $n+2$–BODY PROBLEM WITH AN n–GON BASE

Montserrat Corbera

Departament de Tecnologies Digitals i de la Informació
Universitat de Vic
C/. Laura, 13
08500 Vic, Barcelona, Catalonia, Spain

Jaume Llibre

Departament de Matemàtiques
Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Catalonia, Spain

Abstract. In this paper we prove the existence of central configurations of the $n+2$–body problem where n equal masses are located at the vertices of a regular n–gon and the remaining 2 masses, which are not necessarily equal, are located on the straight line orthogonal to the plane containing the n–gon passing through its center. Here this kind of central configurations is called *bi–pyramidal central configurations*. In particular, we prove that if the masses m_{n+1} and m_{n+2} and their positions satisfy convenient relations, then the configuration is central. We give explicitly those relations.

1. **Introduction.**

We consider the spatial N–body problem

$$m_k\,\ddot{\mathbf{q}}_k = -\sum_{\substack{j\,=\,1\\ j\,\neq\,k}}^{N} G\,m_k\,m_j\,\frac{\mathbf{q}_k-\mathbf{q}_j}{|\mathbf{q}_k-\mathbf{q}_j|^3}\ ,$$

$k=1,\ldots,N$, where $\mathbf{q}_k\in\mathbb{R}^3$ is the position vector of the punctual mass m_k in an inertial coordinate system, and G is the gravitational constant which can be taken equal to one by choosing conveniently the unit of time. The *configuration space* of the spatial N–body problem is

$$\mathcal{E}=\{(\mathbf{q}_1,\ldots,\mathbf{q}_N)\in\mathbb{R}^{3N}\ :\ \mathbf{q}_k\neq\mathbf{q}_j,\ \text{for } k\neq j\}.$$

Given $m_1,\ldots,m_N$ a configuration $(\mathbf{q}_1,\ldots,\mathbf{q}_N)\in\mathcal{E}$ is *central* if there exists a positive constant λ such that

$$\ddot{\mathbf{q}}_k=-\lambda\,(\mathbf{q}_k-\mathbf{c})\ ,$$

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