Applied Mathematics and Computation 246 (2014) 121–147

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Central configurations of the 4-body problem with masses $m_1 = m_2 > m_3 = m_4 = m > 0$ and m small

Montserrat Corbera [a,*], Jaume Llibre [b]

[a] Departament de Tecnologies Digitals i de la Informació, Escola Politcnica Superior, Universitat de Vic – Universitat Central de Catalunya (UVic–UCC), C. de la Laura, 13, 08500 Vic, Catalonia, Spain

[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

ARTICLE INFO

Keywords:
4-body problem
Central configurations
Two-small masses
Convex central configurations
Trapezoidal central configurations

ABSTRACT

In this paper we give a complete description of the families of central configurations of the planar 4-body problem with two pairs of equals masses and two equal masses sufficiently small. In particular, we give an analytical proof that this particular 4-body problem has exactly 34 different classes of central configurations. Moreover for this problem we prove the following two conjectures: There is a unique convex planar central configuration of the 4-body problem for each ordering of the masses in the boundary of its convex hull, which appears in Albouy and Fu (2007) [3]. We also prove the conjecture: There is a unique convex planar central configuration having two pairs of equal masses located at the adjacent vertices of the configuration and it is an isosceles trapezoid. Finally, the families of central configurations of this 4-body problem are numerically continued to the 4-body problem with four equal masses.

1. Introduction and statement of the main results

We consider the planar N-body problem

$$m_k \ddot{\mathbf{q}}_k = -\sum_{\substack{j=1\\ j\neq k}}^{N} G m_k m_j \frac{\mathbf{q}_k - \mathbf{q}_j}{|\mathbf{q}_k - \mathbf{q}_j|^3},$$

$k = 1, \ldots, N$, where $\mathbf{q}_k \in \mathbb{R}^2$ is the position vector of the punctual mass m_k in an inertial coordinate system and G is the gravitational constant which can be taken equal to one by choosing conveniently the unit of time. The *configuration space* of the planar N-body problem is

$$\mathcal{E} = \{(\mathbf{q}_1, \ldots, \mathbf{q}_N) \in \mathbb{R}^{2N} : \mathbf{q}_k \neq \mathbf{q}_j, \text{ for } k \neq j\}.$$

Given $m_1, \ldots, m_N$ a configuration $(\mathbf{q}_1, \ldots, \mathbf{q}_N) \in \mathcal{E}$ is *central* if the acceleration vector for each body is a common scalar multiple of its position vector (with respect to the center of mass). That is, if there exists a positive constant λ such that

$$\ddot{\mathbf{q}}_k = -\lambda(\mathbf{q}_k - \mathbf{cm})$$

* Corresponding author.
E-mail addresses: montserrat.corbera@uvic.cat (M. Corbera), jllibre@mat.uab.cat (J. Llibre).

http://dx.doi.org/10.1016/j.amc.2014.07.109
0096-3003/