CENTRAL CONFIGURATIONS OF THE PLANAR COORBITAL SATELLITE PROBLEM

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Abstract. We study the planar central configurations of the 1 + n body problem where one mass is large and the other n masses are infinitesimal and equal. We find analytically all these central configurations when $2 \le n \le 4$. Numerically, first we provide evidence that when $n \ge 9$ the only central configuration is the regular n-gon with the large mass in its barycenter, and second we provide also evidence of the existence of an axis of symmetry for every central configuration.

Key words: 1 + n body problem, contol configuration, coorbital satellites

1. Introduction

A very old problem in Celestial Mechanics is the study of central configurations of the N-body problem. Central configurations are configurations such that the total Newtonian acceleration of every body is equal to a constant multiplied by the position vector of this body with respect to the center of mass of the configurations.

There is an extensive literature concerning these solutions. For a classical background, see the sections on central configurations in (Wintner, 1941; Hagihara, 1970). For a modern background one can see Smale (1970a, b) and Saari (1980). More recent work can be found in Moulton (1910), Brumberg (1957), Shub (1970), Saari (1980), Schmidt (1980), Saari and Hulkower (1981), Moeckel (1985, 1990, 1994), Perko and Walter (1985), Meyer (1987), Elmabsout (1988), Meyer and Schmidt (1988a, b), Salo and Yoder (1988), Cedó and Llibre (1989), Buck (1991, 1990), Xia (1991), Casasayas et al. (1994), Albouy (1996, 1998), Albouy and Chenciner (1998), Chenciner (1998), Olver (1999), Roberts (2000), Albouy and Llibre (2002). One of the reasons why central configurations are interesting is that they allow to obtain explicit homographic solutions of the N-body problem. This was already



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