# EQUILIBRIUM POINTS AND CENTRAL CONFIGURATIONS FOR THE LENNARD-JONES 2- AND 3-BODY PROBLEMS

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**Abstract.** In this paper we study the relative equilibria and their stability for a system of three point particles moving under the action of a Lennard-Jones potential. A central configuration is a special position of the particles where the position and acceleration vectors of each particle are proportional, and the constant of proportionality is the same for all particles. Since the Lennard-Jones potential depends only on the mutual distances among the particles, it is invariant under rotations. In a rotating frame the orbits coming from central configurations become equilibrium points, the relative equilibria. Due to the form of the potential, the relative equilibria depend on the size of the system, that is, depend strongly of the momentum of inertia $I$. In this work we characterize the relative equilibria, we find the bifurcation values of $I$ for which the number of relative equilibria is changing, we also analyze the stability of the relative equilibria.

**Key words:** central configurations, relative equilibria, spectral stability

## 1. Introduction

In order to get an accurate model to study the action of the intermolecular and gravitational forces at the same time, many authors from physics, astrophysics, astronomy and chemistry have introduced new kinds of potentials, with a structure different from the classical Newtonian's and Coulombian's potentials. In this way, one potential that has been used very often in those branches of the science is the Lennard-Jones potential, which is the topic studied in this paper. For instance, it is used to model the nature and stability of small clusters of interacting particles in crystal growth, random geometry of liquids and in the theory of homogeneous nucleation (see Hoare and Pal, 1971; Wales and Doye, 1997). This potential also appears in molecular dynamics to simulate many particle systems ranging from solids,

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