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Symmetric periodic orbits near a heteroclinic loop formed by two singular points and their invariant manifolds of dimension 1 and 2

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Abstract

In this paper we consider vector fields in $\mathbb{R}^3$ that are invariant under a suitable symmetry and that possess a 'generalized heteroclinic loop' $\mathcal{L}$ formed by two singular points (e^+ and e^-) and their invariant manifolds: one of dimension 2 (a sphere minus the points e^+ and e^-) and one of dimension 1 (the open diameter of the sphere having endpoints e^+ and e^-). In particular, we analyse the dynamics of the vector field near the heteroclinic loop $\mathcal{L}$ by means of a convenient Poincaré map, and we prove the existence of infinitely many symmetric periodic orbits near $\mathcal{L}$. We also study two families of vector fields satisfying this dynamics. The first one is a class of quadratic polynomial vector fields in $\mathbb{R}^3$, and the second one is the charged rhomboidal four-body problem.

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1. Introduction

In this paper we study the periodic motion around a generalized heteroclinic loop $\mathcal{L}$ formed by a two-dimensional sphere $\mathbb{S}^2$ and an interior diameter Γ of the sphere, see figure 1. We suppose that the flow of a system X having such a loop is defined on the closed ball $\mathbb{D}^3$ of $\mathbb{R}^3$ having as boundary $\mathbb{S}^2$. On $\mathbb{S}^2$ we have two foci, e^+ and e^-, diametrally opposite at the endpoints of the diameter Γ. Every orbit on $\mathbb{S}^2$ different from the two foci starts spiraling at e^- and ends spiraling at e^+. In fact, $\mathbb{S}^2\backslash\{e^+, e^-\}$ is the two-dimensional unstable manifold of e^- which coincides with the two-dimensional stable manifold of e^+. Moreover, the diameter Γ is formed by a unique orbit starting at e^+ and ending at e^-; i.e. Γ is the one-dimensional