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Spatial bi-stacked central configurations formed by two dual regular polyhedra

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ABSTRACT

In this paper we prove the existence of two new families of spatial stacked central configurations, one consisting of eight equal masses on the vertices of a cube and six equal masses on the vertices of a regular octahedron, and the other one consisting of twenty masses at the vertices of a regular dodecahedron and twelve masses at the vertices of a regular icosahedron. The masses on the two different polyhedra are in general different. We note that the cube and the octahedron, the dodecahedron and the icosahedron are dual regular polyhedra. The tetrahedron is itself dual. There are also spatial stacked central configurations formed by two tetrahedra, one and its dual.

1. Introduction

We consider the spatial N-body problem

$$m_k\,\ddot{\mathbf{q}}_k = -\sum_{\substack{j=1\\ j\neq k}}^{N} G\,m_k m_j \frac{\mathbf{q}_k-\mathbf{q}_j}{|\mathbf{q}_k-\mathbf{q}_j|^3},$$

$k=1,\dots,N$, where $\mathbf{q}_k\in\mathbb{R}^3$ is the position vector of the punctual mass m_k in an inertial coordinate system, and G is the gravitational constant which can be taken equal to one by choosing conveniently the unit of time. We fix the center of mass $\sum_{i=1}^N m_i\mathbf{q}_i/\sum_{i=1}^N m_i$ of the system at the origin of $\mathbb{R}^{3N}$. The *configuration space* of the N-body problem in $\mathbb{R}^3$ is

$$\mathcal{E} = \left\{(\mathbf{q}_1,\dots,\mathbf{q}_N)\in\mathbb{R}^{3N}\colon \sum_{i=1}^N m_i\mathbf{q}_i = 0,\ \mathbf{q}_i\neq\mathbf{q}_j \text{ for } i\neq j\right\}.$$

Given $m_1,\dots,m_N$ a configuration $(\mathbf{q}_1,\dots,\mathbf{q}_N)\in\mathcal{E}$ is *central* if there exists a positive constant λ such that

$$\ddot{\mathbf{q}}_k = -\lambda\,\mathbf{q}_k,$$

$k=1,\dots,N$. Thus a central configuration $(\mathbf{q}_1,\dots,\mathbf{q}_N)\in\mathcal{E}$ of the N-body problem with positive masses $m_1,\dots,m_N$ is a solution of the system of the N vectorial equations with $N+1$ unknowns ($\mathbf{q}_k$ for $k=1,\dots,N$ plus $\lambda>0$)

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