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ORIGINAL ARTICLE

Periodic motion in perturbed elliptic oscillators revisited

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Abstract We analytically study the Hamiltonian system in $\mathbb{R}^4$ with Hamiltonian

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \frac{1}{2}\left(\omega_1^2 x^2 + \omega_2^2 y^2\right) - \varepsilon V(x, y)$$

being $V(x, y) = -(x^2 y + a x^3)$ with $a \in \mathbb{R}$, where ε is a small parameter and ω_1 and ω_2 are the unperturbed frequencies of the oscillations along the x and y axis, respectively. Using averaging theory of first and second order we analytically find seven families of periodic solutions in every positive energy level of H when the frequencies are not equal. Four of these seven families are defined for all $a \in \mathbb{R}$ whereas the other three are defined for all $a \neq 0$. Moreover, we provide the shape of all these families of periodic solutions. These Hamiltonians may represent the central parts of deformed galaxies and thus have been extensively used and studied mainly numerically in order to describe local motion in galaxies near an equilibrium point.

Keywords Galactic potential · Periodic solutions · Averaging theory

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1 Introduction and statement of the main results

After equilibrium points the periodic solutions are the most simple non-trivial solutions of a differential system. Their study is of special interest because the motion in their neighborhood can be determined by their kind of stability. The stable periodic orbits explain the dynamics of bounded regular motion, while the unstable ones helps to understand the possible chaotic motion of the system. So, periodic orbits play a very important role in understanding the orbital structure of a dynamical system.

Over the last half century dynamical systems perturbing a harmonic oscillator have been used extensively to describe the local motion, i.e. motion near an equilibrium point. The study of this motion have been made mainly using several numerical techniques, see for instance Barbanis (1990), Caranicolas (2001, 2004), Caranicolas and Karanis (1999), Caranicolas and Zotos (2012), Contopoulos (1970b), Elipe et al. (1995), Giorgilli and Galgani (1978), Hénon and Heiles (1964), Karanis and Vozikis (2007), Zotos (2012a,b) to cite just a few.

The general form of a potential for a two-dimensional dynamical system composed of two harmonic oscillators with cubic perturbing terms is

$$U = \frac{1}{2}\left(\omega_1^2 x^2 + \omega_2^2 y^2\right) + \varepsilon V(x, y),$$

where ω_1 and ω_2 are the unperturbed frequencies of the oscillator along the x and the y axes, respectively, ε is the small perturbation parameter and V is the cubic function containing the perturbed terms. We will use the perturbation function

$$V(x, y) = -\left(x^2 y + a x^3\right), \tag{1}$$

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