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TOPOLOGICALLY ANOSOV PLANE HOMEOMORPHISMS

Gonzalo Cousillas — Jorge Groisman — Juliana Xavier

Abstract. This paper deals with classifying the dynamics of *topologically Anosov* plane homeomorphisms. We prove that a topologically Anosov homeomorphism $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ is conjugate to a homothety if it is the time one map of a flow. We also obtain results for the cases when the nonwandering set of f reduces to a fixed point, or if there exists an open, connected, simply connected proper subset U such that $\overline{f(U)} \subset \operatorname{Int}(U)$, and such that

$$\bigcup_{n\leq 0} f^n(U) = \mathbb{R}^2.$$

In the general case, we prove a structure theorem for the α-limits of orbits with empty ω-limit (or the ω-limits of orbits with empty α-limit).

1. Introduction

A homeomorphism $f\colon M \to M$ of the metric space to itself is called *expansive* if there exists $\alpha > 0$ such that given $x, y \in M, x \neq y$, then $d(f^n(x), f^n(y)) > \alpha$ for some $n \in \mathbb{Z}$. The number α is called the *expansivity constant* of f.

The study of expansive systems is both classic and fascinating. In Lewowicz's words [10], the fact that every point has a distinctive dynamical meaning implies that a rich interaction between dynamics and topology is to be expected.

If $\delta > 0$, a δ-pseudo-orbit for f is a sequence $(x_n)_{n\in\mathbb{Z}}$ such that $d(f(x_n), x_{n+1})$ is less than δ for all $n \in \mathbb{Z}$. If $\varepsilon > 0$, we say that the orbit of x ε-shadows a given pseudo-orbit if $d(x_n, f^n(x)) < \varepsilon$ for all $n \in \mathbb{Z}$. Finally, we say that f has the

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