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A Bendixon–Dulac theorem for some piecewise systems

Leonardo P C da Cruz[1] **and Joan Torregrosa**[2,3]

[1] Departamento de Matemática, Universidade Federal de São Carlos, Rodovia Washington Luís, Km 235, Caixa Postal 676, 13565-905 São Carlos, SP, Brazil
[2] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

E-mail: leonardo@mat.uab.cat and torre@mat.uab.cat

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Abstract

The Bendixson–Dulac theorem provides a criterion to find upper bounds for the number of limit cycles in analytic differential systems. We extend this classical result to some classes of piecewise differential systems. We apply it to three different Liénard piecewise differential systems $\ddot{x}+f^{\pm}(x)\dot{x}+x=0$. The first is linear, the second is rational and the last corresponds to a particular extension of the cubic van der Pol oscillator. In all cases, the systems present regions in the parameter space with no limit cycles and others having at most one.

Keywords: piecewise vector field, uniqueness of limit cycle, Bendixson–Dulac theory
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(Some figures may appear in colour only in the online journal)

1. Introduction

The study of the number of isolated periodic orbits, the so called *limit cycles*, is a very relevant problem in the qualitative theory of differential equations. This question, which appears in the second part of the 16th Hilbert problem, was proposed by Hilbert in a list of 23 problems in the International Congress of Mathematics in 1900. In his opinion the study of them would motivate advances in mathematics during the 20th century. In fact, the 16th Hilbert problem is one of the few that remain open, see [20]. This problem has been also reformulated as the

[3] Author to whom any correspondence should be addressed.

