THE GLOBAL FLOW FOR THE SYNODICAL SPATIAL KEPLER PROBLEM

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1 Introduction

The knowledge of the global flow of the synodical spatial circular restricted three–body problem for $\mu = 0$ (i.e. the synodical spatial Kepler problem) allows to describe the flow for $\mu > 0$ sufficiently small during a finite time. The description of this flow for $\mu = 0$ uses the integrability of the system as a Hamiltonian system.

2 Topological study of Hamiltonian systems

It is well known that a Hamiltonian system has at least one first integral, the Hamiltonian H. Every first integral F defines, through their inverse image $F^{-1}(f) = I_f$, sets which are invariant under the flow of the system.

The topological study of a Hamiltonian system which has only one independent first integral is the topological study of the map H; more precisely, the topological classification of the sets I_h and of the points $h \in \mathbb{R}$ for which the topology of I_h changes.

If the Hamiltonian system has more than one independent first integral that are in involution, then we study the topology of the intersections of the sets defined by the inverse image of the first integrals and the foliation of the phase space by these sets. In other words, for a Hamiltonian system with n degrees of freedom, we study the topology of the subsets I_h, I_{h,f_1}, ..., $I_{h,f_1,\dots,f_{r-1}}$ of the phase space where $H, F_1, \dots, F_{r-1}$ are r independent first integrals in involution. In short, we describe how the phase space is foliated by the energy levels I_h and also by the subsequent levels defined by the other first integrals F_i, $i = 1, \dots, r-1$. Of course, $r = n$ if the Hamiltonian system