Qual. Theory Dyn. Syst. (2009) 8:45–205
DOI 10.1007/s12346-009-0002-0

Qualitative Theory of Dynamical Systems

The Global Phase Space for the 2- and 3-Dimensional Kepler Problems

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Received: 5 December 2008 / Accepted: 10 May 2009 / Published online: 17 November 2009

Abstract We determine the foliations of the phase space of four particular integrable Hamiltonian systems obtained from the Kepler problem, namely the sidereal and the synodical Kepler Problem in the plane ($\mathbb{R}^2$) and in the space ($\mathbb{R}^3$). These problems differ in their formulation by the choice of the referentials and by the dimension of the phase space. These four Kepler problems have played a main role in Celestial Mechanics. Their importance is justified: First, the study of an integrable problem allow us to obtain information about a non-integrable problem sufficiently close to the integrable one. In fact this is the principle of perturbation theory. Second, from the point of view of the applications, the sidereal is basic for the computation of the planetary ephemerides and the synodical is the limit case of the non-integrable restricted circular 3-body problem when one of the masses of the two primaries tends to zero. We determine the foliations of the phase space of these four Kepler problems by the orbits (i.e. we characterize their global flow), and by fixing one, two or three independent first integrals in involution; of course, at most three for the two spatial problems, and at most two for the two planar problems.

Keywords Planar Kepler problem · Spacial Kepler problem · Global flow

Mathematics Subject Classification (2000) Primary 37C10; Secondary 37C15

M. P. Dantas was supported by a CAPES grant. J. Llibre was supported by a MEC/FEDER grant number MTM2008-03437 and by a CICYT grant number 2009SGR 410.

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