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Cyclicity of a fake saddle inside the quadratic vector fields

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Abstract

This paper concerns the study of small-amplitude limit cycles that appear in the phase portrait near an unfolded fake saddle singularity. This degenerate singularity is also known as an impassable grain. The canonical form of the unperturbed vector field is like a degenerate flow box. Near the singularity, the phase portrait consists of parallel fibers, all but one of which have no singular points, and at the singular fiber, there is one node. We demonstrate different techniques in order to show that the cyclicity is bigger than or equal to two when the canonical form is quadratic.

MSC: 34C07; 37G10; 37G15; 34D15

Keywords: Cyclicity; Fake saddle; Impassable grain; Limit cycle; Bifurcation; Singularity unfolding

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