POINCARÉ COMPACTIFICATION OF HAMILTONIAN POLYNOMIAL VECTOR FIELDS*

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Introduction. There exists an extensive literature on changes of variables which transform the equations of motion of interesting problems in Celestial Mechanics into polynomial form (see [Heg]). In most cases this is achieved by regularizing double collisions or introducing redundant variables, or both. In previous works [DLLP1,2] we have exploited this idea to extend the equations of motion of the collinear three body problem to a compact manifold by means of the Poincaré compactification. Such compactification was introduced to study the behaviour at infinity of polynomial vector fields (see for instance [CL]). In the Poincaré compactification of some n-body problems the critical points which appear there are extremely degenerate. In this paper we focus our attention on generic properties of arbitrary Hamiltonian polynomial vector fields, especially at infinity.

In the first Section, we review the Poincaré compactification for a general polynomial vector field. Since this is fundamental for our computations, we present all the details with a slight modification in the notation from the construction outlined in [CL].

In Section 2, we give the global expressions for the Poincaré compactification of a Hamiltonian polynomial vector field. The explicit computations of the expressions of the vector field in local charts are also given. All results from Section 3 to 5 refer to the Poincaré compactification of Hamiltonian polynomial vector fields.

Section 3 summarizes the behavior of the vector field at infinity. We see that all the energy levels extend to the same invariant set at infinity. Section 4 is devoted to generic properties. We prove that generically, in the coefficient topology, the above invariant set at infinity is a smooth manifold and all the critical points in the Poincaré sphere are hyperbolic. In the final section, we study two simple cases of Hamiltonian polynomial vector fields. A fairly complete description of the flow can be given in these two cases.

We remark that the majority of Hamiltonian polynomial vector fields appearing in Classical Mechanics, like for example the Henon-Heiles and Contopoulos, are non generic in the above sense.

1. Poincaré compactification for polynomial vector fields. As far as we know the Poincaré compactification for a polynomial vector field

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