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SIERPIŃSKI CURVE JULIA SETS FOR QUADRATIC RATIONAL MAPS

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Abstract. We investigate under which dynamical conditions the Julia set of a quadratic rational map is a Sierpiński curve.

1. Introduction

Iteration of rational maps in one complex variable has been widely studied in recent decades continuing the remarkable papers of Fatou and Julia who introduced normal families and Montel's Theorem to the subject at the beginning of the twentieth century. Indeed, these maps are the natural family of functions when considering iteration of holomorphic maps on the Riemann sphere $\widehat{\mathbf{C}}$. For a given rational map f, the sphere splits into two complementary domains: the *Fatou set* $\mathcal{F}(f)$ where the family of iterates $\{f^n(z)\}_{n\geq 0}$ forms a normal family, and its complement, the *Julia set* (f). The Fatou set, when non-empty, is given by the union of, possibly, infinitely many open sets in $\widehat{\mathbf{C}}$, usually called Fatou components. On the other hand, it is known that the Julia set is a closed, totally invariant, perfect non-empty set, and coincides with the closure of the set of (repelling) periodic points. For background see [7].

Unless the Julia set of f fills up the whole sphere, one of the major problems in complex dynamics is to characterize the topology of the Julia set (or at least determine some topological properties) and, if possible, study the chaotic dynamics on this invariant set when iterating the map. Indeed, depending on f, the Julia set can have either trivial topology (for instance just a circle), or a highly rich topology (for instance it may be a non locally connected continuum, a dendrite, a Cantor set, a Cantor set of circles, etc.)

The Sierpiński carpet fractal shown in Figure 1 is one of the best known planar, compact and connected sets. On the one hand, it is a *universal plane continuum* in the sense that it contains a homeomorphic copy of any planar, one-dimensional, compact and connected set. On the other hand, there is a topological characterization of this

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