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When Parallels and Meridians are Limit Cycles for Polynomial Vector Fields on Quadrics of Revolution in the Euclidean 3-Space

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This article is dedicated to Professor Mauricio Matos Peixoto on his 95th birthday.

We study polynomial vector fields of arbitrary degree in $\mathbb{R}^3$ with an invariant quadric of revolution. We characterize all the possible configurations of invariant meridians and parallels that these vector fields can exhibit. Furthermore, we analyze when these invariant meridians and parallels can be limit cycles.

Keywords: Polynomial vector field; invariant parallel; invariant meridian; limit cycle; periodic orbit.

1. Introduction and Statement of the Results

One of the most difficult objects to control in the qualitative theory of ordinary differential equations in dimension two are the limit cycles, their maximum number and their distribution. This problem is related to the second part of the 16th Hilbert problem (see [Hilbert, 1900]) when the differential equations are polynomial.

When the limit cycles are algebraic, that is, they leave on an algebraic curve, the previous questions on the number of limit cycles and their distribution for polynomial differential equations in the plane are already solved for the so-called generic algebraic limit cycles, see [Llibre *et al.*, 2010a, 2010b; Llibre & Rodríguez, 2004; Zhang, 2011].

The main objective of this paper is to extend the study of algebraic limit cycles to polynomial

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