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Polynomial Hamiltonian systems of degree 3 with symmetric nilpotent centers

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Abstract

We provide normal forms and the global phase portraits in the Poincaré disk for all Hamiltonian planar polynomial vector fields of degree 3 symmetric with respect to the x-axis having a nilpotent center at the origin.

Keywords: Polynomial Hamiltonian systems; Nilpotent center; Phase portrait; Poincaré compactification

1. Introduction and statement of the results

Hamiltonian systems are relevant for many physical studies. Let $H(x, y)$ be a real polynomial in the variables x and y. Then a system of the form

$$x' = H_y(x, y) \qquad y' = -H_x(x, y) \tag{1}$$

is called a *polynomial Hamiltonian system*. Here the prime denotes derivative with respect to the independent variable t.

Poincaré in [20] defined a *center* for a vector field on the real plane as a singular point having a neighborhood filled with periodic orbits with the exception of the singular point. Let $p \in \mathbb{R}^2$ be a singular point of an analytic differential system in $\mathbb{R}^2$, and assume that p is a center. Without loss of generality we can assume that p is at the origin of coordinates. Then after a linear change of variables and a rescaling of the time variable (if necessary), the

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