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# $\mathbb{Z}_2$-SYMMETRIC PLANAR POLYNOMIAL HAMILTONIAN SYSTEMS OF DEGREE 3 WITH NILPOTENT CENTERS

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Abstract. We provide the normal forms of all $\mathbb{Z}_2$-symmetric planar polynomial Hamiltonian systems of degree 3 having a nilpotent center at the origin. Furthermore, we complete the classification of the global phase portraits in the Poincaré disk of the above systems initiated by Dias, Llibre and Valls [9].

## 1. Introduction and statement of results

In this article we study the global phase portrait of all $\mathbb{Z}_2$-symmetric planar polynomial Hamiltonian systems of degree 3 having a nilpotent center at the origin. Let $H(x, y)$ be a real polynomial in the variables $x$ and $y$. Then a system of the form

$$x' = H_y \quad y' = -H_x$$

is called a *polynomial Hamiltonian system*. Here the prime denotes derivative with respect to the independent variable $t$.

Poincaré [22] defined a *center* for a vector field on the real plane as a singular point having a neighborhood filled with periodic orbits with the exception of the singular point. Let $p \in \mathbb{R}^2$ be a singular point of an analytic differential system in $\mathbb{R}^2$, and assume that $p$ is a center. Without loss of generality we can assume that $p$ is at the origin of coordinates. Then after a linear change of variables and a rescaling of the time variable (if necessary), the system can be written in one of the following three forms

$$x' = -y + P(x, y), \quad y' = x + Q(x, y), \tag{1.1}$$

$$x' = y + P(x, y), \quad y' = Q(x, y), \tag{1.2}$$

$$x' = P(x, y), \quad y' = Q(x, y), \tag{1.3}$$

where $P(x, y)$ and $Q(x, y)$ are real analytic functions without constant and linear terms, defined in a neighborhood of the origin. In what follows a center of an analytic differential system in $\mathbb{R}^2$ is called *linear type*, *nilpotent* or *degenerate* if after an affine change of variables and a rescaling of the time it can be written as system (1.1), (1.2) or (1.3), respectively.

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