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Boundedness, invariant algebraic surfaces and global dynamics for a spectral model of large-scale atmospheric circulation

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We consider a three-dimensional quadratic system S in $\mathbb{R}^3$ with six parameters which appears in geophysical fluid dynamics (atmospheric blocking). In this paper we start its systematic study from the point of view of dynamical systems. First, we reduce the number of its parameters from six to three. Thus, we must study a three-dimensional quadratic system with three parameters, which recalls us the famous Lorenz-63 system. Traditionally, system S has been studied by considering two subcases, called the conservative and the dissipative case, as the parameter responsible for dissipation is zero or not. In the conservative case, we reduce system S to systems without parameters. Among these there are two interesting systems: one is homeomorphic to the simple pendulum, and the other is a perturbation of it. In the latter system the saddle point corresponding to topographic instability is connected to two homoclinic orbits to it. In the dissipative case we prove that all trajectories of system S enter in an ellipsoid for any values of the parameters. We characterize their invariant algebraic surfaces of degree 2, and for those systems having such invariant algebraic surfaces we describe their global phase portraits. © *2005 American Institute of Physics.* [DOI: 10.1063/1.1955448]

I. INTRODUCTION

In this paper we consider the following quadratic system in $\mathbb{R}^3$

$$\dot{x} = az - b(x - c),$$

$$\dot{y} = -(dx - e)z - by,$$

$$\dot{z} = (dx - e)y - fx - bz, \tag{1}$$

where a, b, c, d, e, and f are arbitrary real parameters and the overdot denotes differentiation with respect to time t. We start the systematic study of its flow from the point of view of dynamical systems. System (1) with positive parameters is a subsystem of Charney DeVore (CdV) model, well known in geophysical fluid dynamics, and describes topographically driven disturbances of an atmospheric zonal flow. The CdV model is a quadratic system in $\mathbb{R}^6$ introduced by Charney and DeVore in 1979 as a truncation of the equations describing barotropic atmospheric flow over topography. They found that more than one stable equilibrium state may occur for a given external

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