SUFFICIENT CONDITIONS FOR BOUNDEDNESS OF THE FLOW OF A DIFFERENTIAL SYSTEM ON $\mathbb{R}^n$

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Abstract. In this paper we provide sufficient conditions for boundedness of the flow of a differential system on $\mathbb{R}^n$. Our result is a generalization of the well–known Liapunov's Stability Theorem. As an application, we prove the boundedness of the flow of some famous polynomial differential systems: Lorenz–63, Lorenz–84, Chen and of a system equivalent to the three–component dissipative Charney–DeVore model of large–scale atmospheric flow over topography.

1. Introduction

In this paper we present a generalization of the well–known Liapunov's Stability Theorem. Our result provides sufficient conditions for the boundedness of the flow of a smooth differential system on $\mathbb{R}^n$. We say that the flow of such a system is *bounded* if all trajectories of the system in forward time remain inside a compact set in $\mathbb{R}^n$. A typical example of bounded flows in Geophysical Fluid Dynamics is given by the following result.

Theorem 1. [8, 4] *Consider the differential system on* $\mathbb{R}^n$

$$dx_i/dt = \dot{x}_i = \sum_{j,k=1}^{n} a_{ijk} x_j x_k + \sum_{j=1}^{n} a_{ij} x_j + a_i, \quad (i = 1, 2, \dots, n), \tag{1}$$

where the coefficients a_{ijk}, a_{ij}, *and* a_i *are real constants. If its coefficients satisfy the following conditions:*

(a) *the polynomial* $\sum_{i,j,k=1}^{n} a_{ijk} x_i x_j x_k$ *in the variables* $x_1, x_2, \dots, x_n$ *is the null polynomial,*

(b) *the quadratic form* $\sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j$ *is negative definite, i.e.* $\sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j < 0$ *for any real* $\xi_1, \xi_2, \dots, \xi_n$ *such that* $(\xi_1, \xi_2, \dots, \xi_n) \neq 0$,

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