DISTINGUISHING BETWEEN LOCAL AND NON–LOCAL DISSIPATIVITY OF VECTOR FIELDS ON $\mathbb{R}^n$

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Abstract. We consider two types of dissipativity for the vector fields on $\mathbb{R}^n$: local or Div–dissipativity, characterized by negative divergence of the vector field, and nonlocal or Ball–dissipativity, defined by the existence of a compact set which absorbs all trajectories of the vector field.

We prove that Div–dissipativity neither implies, nor it is implied by the Ball–dissipativity. To this purpose, we make use of a well–known theorem in Geophysical Fluid Dynamics, proved by Lorenz and later also by Ghil and Childress, which provides sufficient conditions of Ball–dissipativity for quadratic polynomial vector fields on $\mathbb{R}^n$.

Next for quadratic polynomial vector fields we prove that the sufficient conditions of the Lorenz's theorem are not implied by the Ball–dissipativity (i.e. the converse of Lorenz's theorem does not hold), and that these conditions have no relation with the Div–dissipativity. Along the way we state and prove a Liapunov–like result of Ball–dissipativity on $\mathbb{R}^n$. As an application we get that the tridimensional Charney–DeVore system of large scale atmospheric circulation is Ball-dissipative for all values of its parameters.

1. Introduction and statement of results

Consider the differential system

$$\dot{\mathbf{x}} = f(\mathbf{x}), \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is of class C^1. We say that the vector field f or that the differential system (1) is *Div–dissipative*, or locally dissipative, if

$$\operatorname{div} f(\mathbf{x}) < 0 \quad \text{for all} \quad x \in \mathbb{R}^n.$$

Clearly, if the vector field f is Div–dissipative, then volumes in the phase space $\mathbb{R}^n$ contract to zero under the flow of (1) (see e.g. [16]).

Let ϕ_t be the flow of (1). We say that the vector field f or that the differential system (1) is *Ball–dissipative*, or non–locally dissipative, if there exists a compact set $K \subset \mathbb{R}^n$ such that for all $\mathbf{x} \in \mathbb{R}^n$, $\phi_t(\mathbf{x})$ belongs to K for all t sufficiently large (see [14]). This property is known in the literature also under the name of eventually uniformly boundedness of solutions of (1) (see [17]), or ultimately boundedness of solutions of (1) (see [15]). The compact set K, eventually absorbing all trajectories of (1), is a trapping region [17] or a "ball" of dissipation [1].

It is natural to ask what is the relationship between these two types of dissipativity. In the papers [18], [2] and [8], the considered vector fields are that of Chen's system, of a

2000 *Mathematics Subject Classification.* Primary 37C10, Secondary 86A10.

Key words and phrases. polynomial differential system, dissipativity, Liapunov function.

† The first author was partially supported by a Marie Curie grant HPMT–CT–2001–00247, and the second author was partially supported by MCYT grant MTM2005-06098-C02-01 and by a CIRIT grant number 2005SGR 00550.