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Existence of Herman Rings for Meromorphic Functions

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We apply the Shishikura surgery construction to transcendental maps in order to obtain examples of meromorphic functions with Herman rings, in a variety of possible arrangements. We give a sharp bound on the maximum possible number of such rings that a meromorphic function may have, in terms of the number of poles. Finally we discuss the possibility of having "unbounded" Herman rings (i.e., with an essential singularity in the boundary), and give some examples of maps with this property.

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1. INTRODUCTION

Given a map f defined on the complex plane $\mathbb{C}$, the sequence formed by its iterates is denoted by $f^0 := \mathrm{Id}$, $f^n := f \circ f^{n-1}$, $n \in \mathbb{N} = \{1, 2, \ldots\}$. The grand orbit of a point z under $f : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ consists of all points $z' \in \hat{\mathbb{C}}$ whose orbits eventually intersect the orbit of z. It is well known (see e.g. [18]) that for a holomorphic map of the Riemann sphere, a maximum of two points are allowed to have a finite grand orbit. These points are called *exceptional values* and always belong to the stable or Fatou set (see below). We shall denote by $E(f)$ the set of exceptional values of f.

When one considers iteration of a holomorphic self map $f : X \to X$ where X is a Riemann surface, the study makes sense and is non-trivial when X is either the Riemann sphere $\hat{\mathbb{C}}$, the complex plane $\mathbb{C}$ or the complex plane minus one point $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$, sets that should be viewed as $\hat{\mathbb{C}}$ minus one and two points (exceptional

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