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Orbital stability of solitary waves of moderate amplitude in shallow water

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ABSTRACT

We study the orbital stability of solitary traveling wave solutions of an equation for surface water waves of moderate amplitude in the shallow water regime. Our approach is based on a method proposed by Grillakis, Shatah and Strauss (1987) [1], and relies on a reformulation of the evolution equation in Hamiltonian form. We deduce stability of solitary waves by proving the convexity of a scalar function, which is based on two nonlinear functionals that are preserved under the flow.

1. Introduction

This paper is concerned with an equation for surface waves of moderate amplitude in the shallow water regime which arises as an approximation to the Euler equations. Since the exact governing equations for water waves have proven to be nearly intractable (Gerstner waves being the only known explicit solutions to the full equations [2–5]), the quest for suitable simplified model equations was initiated at the earliest stages of the development of hydrodynamics. Evolving from an analysis confined to linear theory which dominated most studies until the early twentieth century [6], many competing nonlinear models are being proposed to this day to gain insight into phenomena like wave breaking or solitary waves. One of the most prominent examples is the Camassa–Holm equation [7], which is an integrable infinite-dimensional Hamiltonian system [8–10] whose solitary waves are solitons [11,12]. Some classical solutions of the Camassa–Holm equation develop singularities in finite time in the form of wave breaking, i.e. the solution remains bounded but its slope becomes unbounded [13]. After blow-up, the solutions recover in the sense of global weak solutions [14,15].

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