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Non-uniform continuity of the flow map for an evolution equation modeling shallow water waves of moderate amplitude

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ABSTRACT

We prove that the flow map associated to a model equation for surface waves of moderate amplitude in shallow water is not uniformly continuous in the Sobolev space H^s with $s > 3/2$. The main idea is to consider two suitable sequences of smooth initial data whose difference converges to zero in H^s, but such that neither of them is convergent. Our main theorem shows that the exact solutions corresponding to these sequences of data are uniformly bounded in H^s on a uniform existence interval, but the difference of the two solution sequences is bounded away from zero in H^s at any positive time in this interval. The result is obtained by approximating the solutions corresponding to these initial data by explicit formulae and by estimating the approximation error in suitable Sobolev norms.

1. Introduction and the main result

We consider a model equation for surface waves of moderate amplitude in shallow water

$$u_t + u_x + 6uu_x - 6u^2u_x + 12u^3u_x + u_{xxx} - u_{xxt} + 14uu_{xxx} + 28u_xu_{xx} = 0, \tag{1}$$

which arises as an approximation of the Euler equations in the context of homogeneous, inviscid gravity water waves. In recent years, several nonlinear models have been proposed in order to understand some important aspects of water waves, like wave breaking or solitary waves. One of the most prominent examples is the Camassa–Holm (CH) equation [1], which is an integrable, infinite-dimensional Hamiltonian system [2–4]. The relevance of the CH equation as a model for the propagation of shallow water waves was discussed by Johnson [5], where it is shown that it describes the horizontal component of the velocity field at a certain depth within the fluid; see also [6]. Building upon the ideas presented in [5], Constantin and Lannes [7] have recently derived the evolution equation (1) as a model for the motion of the free surface of the wave, and they evince that (1) approximates the governing equations to the same order as the CH equation. Besides deriving (1), the authors of [7] also establish the local well-posedness results for the Cauchy problem associated to (1). Relying on a semigroup approach due to Kato [8], Duruk [9] has shown that this feature holds for a larger class of initial data, as well as for solutions which are spatially periodic [10]. The well-posedness in the context of Besov spaces together with the regularity and the persistence properties of strong solutions are studied in [11].

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