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ORIGINAL RESEARCH

Dynamics of a Generalized Rayleigh System

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Abstract
Consider the first order differential system given by

$$\dot{x} = y, \qquad \dot{y} = -x + a(1 - y^{2n})y,$$

where a is a real parameter and the dots denote derivatives with respect to the time t. Such system is known as the generalized Rayleigh system and it appears, for instance, in the modeling of diabetic chemical processes through a constant area duct, where the effect of adding or rejecting heat is considered. In this paper we characterize the global dynamics of this generalized Rayleigh system. In particular we prove the existence of a unique limit cycle when the parameter $a \neq 0$.

Keywords Rayleigh system · Limit cycles · Averaging theory · Poincaré compactification

Mathematics Subject Classification Primary: 34C07 · 34C05 · 37G15

Introduction and Statement of the Main Results

Consider the non-linear second order differential equation

$$\ddot{x} + x = a(1 - \dot{x}^{2n})\dot{x}, \tag{1}$$

where n is a positive integer and $a \in \mathbb{R}$ and the dots denote derivatives with respect to the time t. This equation is equivalent to the following first order differential system

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