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PERIODIC SOLUTIONS OF *EL NIÑO* MODEL THROUGH THE VALLIS DIFFERENTIAL SYSTEM

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Abstract. By rescaling the variables, the parameters and the periodic function of the Vallis differential system we provide sufficient conditions for the existence of periodic solutions and we also characterize their kind of stability. The results are obtained using averaging theory.

1. **Introduction.**

The Vallis system, introduced by Vallis [10] in 1988, is a periodic non–autonomous 3–dimensional system that models the atmosphere dynamics in the tropics over the Pacific Ocean, related to the yearly oscillations of precipitation, temperature and wind force. Denoting by x the wind force, by y the difference of near–surface water temperatures of the east and west parts of the Pacific Ocean, and by z the average near–surface water temperature, the Vallis system is

$$\begin{aligned} \frac{dx}{dt} &= -ax + by + au(t), \\ \frac{dy}{dt} &= -y + xz, \\ \frac{dz}{dt} &= -z - xy + 1, \end{aligned} \tag{1}$$

where $u(t)$ is some C^1 $T-$periodic function that describes the wind force under seasonal motions of air masses, and the parameters a and b are positive.

Although this model neglects some effects like Earth's rotation, pressure field and wave phenomena, it provides a correct description of the observed processes and recovers many of the observed properties of El Niño. The properties of El Niño phenomena are studied analytically in [9] and [10]. More precisely, in [10] it is shown that taking $u \equiv 0$, it is possible to observe the presence of chaos by considering $a = 3$ and $b = 102$. Later on, in [9] it is proved that exists a chaotic attractor for system (1) after a Hopf bifurcation. This chaotic motion can be easily understanding if we observe that there exist a strong similarity between system (1)

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