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On the number of limit cycles in discontinuous piecewise linear differential systems with two pieces separated by a straight line

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ABSTRACT

In this paper we study the maximum number N of limit cycles that can exhibit a planar piecewise linear differential system formed by two pieces separated by a straight line. More precisely, we prove that this maximum number satisfies $2 \leq N \leq 4$ if one of the two linear differential systems has its equilibrium point on the straight line of discontinuity.

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1. Introduction and statement of the main result

The study of piecewise linear differential systems goes back to Andronov, Vitt and Khaikin [1] and still continues to receive attention by researchers. These last years a renewed interest has appeared in the mathematical community working in differential equations for understanding the dynamical richness of the piecewise linear differential systems, because these systems are widely used to model processes appearing in electronics, mechanics, economy, etc., see for instance the books of di Bernardo, Budd, Champneys and Kowalczyk [3], and Simpson [25], and the survey of Makarenkov and Lamb [23], and the hundreds of references quoted in these last three works.

We recall that a *limit cycle* is a periodic orbit of a differential system which is isolated in the set of all periodic orbits of the system.

The simplest possible continuous but nonsmooth piecewise linear differential systems are the ones having only two pieces separated by a straight line. In 1990 Lum and Chua [22] conjectured that a continuous piecewise linear vector field in the plane with two pieces has at most one limit cycle. We note that even in this apparent simple case, only after a difficult analysis it was possible to prove the existence of at most

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