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Sufficient conditions for the existence of periodic solutions of the extended Duffing–Van der Pol oscillator

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In this paper, some aspects on the periodic solutions of the extended Duffing–Van der Pol oscillator are discussed. Doing different rescaling of the variables and parameters of the system associated with the extended Duffing–Van der Pol oscillator, we show that it can bifurcate one or three periodic solutions from a two-dimensional manifold filled by periodic solutions of the referred system. For each rescaling we exhibit concrete values for which these bounds are reached. Beyond that we characterize the stability of some periodic solutions. Our approach is analytical and the results are obtained using the averaging theory and some algebraic techniques.

Keywords: extended Duffing–Van der Pol oscillator; periodic solution; non-autonomous systems; averaging theory

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1. Introduction

1.1 *Setting the problem*

A large number of non-autonomous chaotic phenomena in physics, engineering, mechanics and biology, among others, are described by second-order differential systems of the form

$$\ddot{x} = g(x, \dot{x}, t) + \gamma(t), \tag{1}$$

where $g(x, \dot{x}, t)$ is a continuous function and $\gamma(t)$ is some external force. For instance, in biology, system (1) models the FHN neuron oscillator, and in engineering, system (1) is a model to the horizontal platform system.

The specific topic addressed in this paper concerns another particular case of Equation (1), namely, an extension of the forced Van der Pol equation with external excitation. Van der Pol's system also plays an important role in many applications in areas such as engineering, biology, physics and seismology (see [2] and references therein).

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