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Singularities of inner functions associated with hyperbolic maps

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ABSTRACT

Let f be a function in the Eremenko-Lyubich class $\mathcal{B}$, and let U be an unbounded, forward invariant Fatou component of f. We relate the number of singularities of an inner function associated to $f|_U$ with the number of tracts of f. In particular, we show that if f lies in either of two large classes of functions in $\mathcal{B}$, and also has finitely many tracts, then the number of singularities of an associated inner function is at most equal to the number of tracts of f. Our results imply that for hyperbolic functions of finite order there is an upper bound – related to the order – on the number of singularities of an associated inner function.

1. Introduction

Let f be a transcendental entire function. We denote by f^n the nth iterate of f. The set of points for which the iterates $\{f^n\}_{n\in\mathbb{N}}$ form a normal family in some neighbourhood is called the *Fatou set* $F(f)$. Its complement in the complex plane is the *Julia set* $J(f)$. The Fatou set is open and consists of connected components which are called *Fatou components*. For an introduction to these sets and their properties see [10].

Let $U \subset \mathbb{C}$ be an unbounded *forward invariant* Fatou component; in other words, $f(U) \subset U$. Note that it follows from [1, Theorem 3.1] that U is simply connected. Let also $\phi : \mathbb{D} \to U$ be a Riemann map. Then the function $h : \mathbb{D} \to \mathbb{D}$ defined by $h := \phi^{-1} \circ f \circ \phi$ is an *inner function associated to* $f|_U$. Note that h is unique up to a conformal conjugacy.

A point $\zeta \in \partial\mathbb{D}$ is a *singularity* of h if h cannot be extended holomorphically to any neighbourhood of ζ in $\mathbb{C}$. This definition is independent of the choice of ϕ, up to a Möbius map (see Section 2.1). In this paper we are interested in the number of singularities of an associated inner function h. In particular, we give two

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