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Surgery on the limbs of the Mandelbrot set

Introduction

The technique of *quasiconformal surgery* on complex polynomials as a way of constructing certain maps between parameter spaces was first introduced in [BD] to relate a certain cubic parameter space to the 1/2–limb of the Mandelbrot set. Afterwards, surgery was used in [BF] to construct homeomorphisms between some limbs of the Mandelbrot set. Recently, it has been announced in [EY] the existence of products of Mandelbrot sets in the two–dimensional complex parameter space of cubic polynomials, also by means of surgery.

The goal of this paper is to outline the results and proofs published in [BF] and presented at the *Göttingen Workshop on Siegel Disks* on December 1995. They are presented hoping that the reader can realize the important points in the proofs without entering into the many details that are necessary to formalize the surgery process. We assume familiarity with the basic definitions and concepts in complex dynamics, although we give a brief summary of the necessary ones to state the main results. From that point on, we refer to the adapted introduction in [BF] where the main tools are summarized, or to the original sources contained in the bibliography. Finally, Sections 4 and 5 contain some new material on further results (in progress) and some conjectures and questions on related topics.

1 Preliminaries and main results

For a complex polynomial f of degree $d > 1$, the point at infinity is always a superattracting fixed point. Since infinity has no other preimage than itself, its basin of attraction is connected. The complement of this basin is called the *filled Julia set of* f, that is,

$$K(f) = \{z \in \mathbb{C} \mid \{f^n(z)\}_n \text{ is bounded}\}.$$

The set $K(f)$ is compact and simply connected by the observations above. Its connectedness though, depends on the orbits of the *critical points* of f, or zeros of f'. Indeed, $K(f)$ is connected if and only if all critical points of f belong to $K(f)$ (see, for example, [Bl]).

We define the *Julia set* of f, denoted by $J(f)$, as the common boundary of $K(f)$ and the basin of attraction of infinity. This definition only works for polynomials. In a general setting, the Julia set is the set of points for which the family of iterates does not form a normal family.

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