DYNAMICS OF THE COMPLEX STANDARD FAMILY

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ABSTRACT. The complexification of the standard family of circle maps $\theta \mapsto \theta + \alpha + \beta \sin(\theta) \bmod (2\pi)$, whose parameter space contains the well-known Arnold tongues, is given by $F_{\alpha\beta}(\omega) = \omega e^{i\alpha} e^{(\beta/2)(\omega - 1/\omega)}$, a holomorphic map of $\mathbb{C}^*$ with essential singularities at 0 and ∞. For real values of the parameters, we study the dynamical plane of the family $F_{\alpha\beta}$. Near the essential singularities we prove the existence of *hairs* in the Julia set, an invariant set of curves organized by some symbolic dynamics, and whose points (that are not endpoints) tend exponentially fast to 0 or ∞ under iteration. For $\beta < 1$, we give a complex interpretation of the bifurcations of the family of circle maps. More precisely, we give a new characterization of the rational Arnold tongues in terms of some of the hairs attaching to the unit circle. For certain irrational rotation numbers, we show that the Fatou set consists exclusively of a Herman ring and its preimages. For $\beta > 1$ we prove that, under certain conditions, all hairs end up attached to the unit circle as we increase the parameter.

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