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CAPTURE ZONES OF THE FAMILY OF FUNCTIONS $\lambda z^m \exp(z)$

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We consider the family of entire transcendental maps given by $F_{\lambda,m}(z) = \lambda z^m \exp(z)$ where $m \geq 2$. All functions $F_{\lambda,m}$ have a superattracting fixed point at $z = 0$, and a critical point at $z = -m$. In the dynamical plane we study the topology of the basin of attraction of $z = 0$. In the parameter plane we focus on the capture behavior, i.e. λ values such that the critical point belongs to the basin of attraction of $z = 0$. In particular, we find a capture zone for which this basin has a unique connected component, whose boundary is then nonlocally connected. However, there are parameter values for which the boundary of the immediate basin of $z = 0$ is a quasicircle.

Keywords: Iteration; entire functions; Julia set; Fatou set; polynomial-like mappings; locally connected set.

1. Introduction

Our goal in this paper is to study some dynamical aspects of the families of entire transcendental maps

$$F_{\lambda,m}(z) = \lambda z^m \exp(z)\,, \quad m \geq 2\,.$$

Observe that $m = 0$ corresponds to the exponential family $E_\lambda(z) = \lambda \exp(z)$, the simplest example of an entire transcendental map with a unique asymptotic value, $z = 0$, in analogy with the well-known quadratic family of polynomials $z \to z^2 + c$. The exponential map has been thoroughly studied by many authors (see e.g. [Devaney & Krych, 1984; Devaney & Tangerman, 1986]).

The case $m = 1$ corresponds to $G_\lambda(z) = \lambda z \exp(z)$ which appeared for the first time in [Baker, 1970] as an example of an entire transcendental map whose Julia set is the whole plane (for an appropriate value of λ). Later on, this family was studied in [Fagella, 1995; Geyer, 2001]. The asymptotic value $z = 0$ of $G_\lambda(z)$ is fixed and its multiplier is $G'_\lambda(0) = \lambda$. Hence its dynamical character depends on the parameter λ. Besides this point, the dynamical behavior of G_λ is determined by the orbit of the critical point $z = -1$.

Some functions in the family $F_{\lambda,m} = \lambda z^m \exp(z)$ for $m \geq 2$ have been used in the literature as examples of certain dynamical phenomena (see e.g. [Bergweiler, 1995], for a Baker domain at a

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