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Surgery on Herman rings of the complex standard family

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Abstract. We consider the standard family (or Arnold family) of circle maps given by $f_{\alpha,\beta}(x) = x + \alpha + \beta \sin(x) \pmod{2\pi}$, for $x, \alpha \in [0, 2\pi)$, $\beta \in (0, 1)$ and its complexification $F_{\alpha,\beta}(z) = ze^{i\alpha} \exp[\frac{1}{2}\beta(z - \frac{1}{z})]$. If $f_{\alpha,\beta}$ is analytically linearizable, there is a Herman ring around the unit circle in the dynamical plane of $F_{\alpha,\beta}$. Given an irrational rotation number θ, the parameters (α, β) such that $f_{\alpha,\beta}$ has rotation number θ form a curve T_θ in the parameter plane. Using quasi-conformal surgery of the simplest type, we show that if θ is a Brjuno number, the curve T_θ can be parametrized real-analytically by the modulus of the Herman ring, from $\beta = 0$ up to a point (α_0, β_0) with $\beta_0 \le 1$, for which the Herman ring collapses. Using a result of Herman and a construction in I. N. Baker and P. Domínguez (*Complex Variables* **37** (1998), 67–98) we show that for a certain set of angles $\theta \in \mathcal{B} \setminus \mathcal{H}$, the point β_0 is strictly less than 1 and, moreover, the boundary of the Herman rings with the corresponding rotation number have two connected components which are quasi-circles, and do not contain any critical point. For rotation numbers of constant type, the boundary consists of two quasi-circles, each containing one of the two critical points of $F_{\alpha,\beta}$.

1. *Introduction*

The standard family of maps of the circle is a two-parameter family given by

$$f_{\alpha,\beta}(x) = x + \alpha + \beta \sin(x) \qquad \pmod{2\pi},$$

for $x, \alpha \in [0, 2\pi)$ and $\beta \in (0, 1)$. These maps are simple perturbations of rigid rotations and it is well understood how their dynamics vary in terms of the parameters α and β.