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Wandering domains for composition of entire functions ☆

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ABSTRACT

C. Bishop in [10, Theorem 17.1] constructs an example of an entire function f in class $\mathcal{B}$ with *at least* two grand orbits of oscillating wandering domains. In this paper we show that his example has *exactly* two such orbits, that is, f has no *unexpected* wandering domains. We apply this result to the classical problem of relating the Julia sets of composite functions with the Julia set of its members. More precisely, we show the existence of two entire maps f and g in class $\mathcal{B}$ such that the Fatou set of $f \circ g$ has a wandering domain, while all Fatou components of f or g are preperiodic. This complements a result of A. Singh in [22, Theorem 4] and results of W. Bergweiler and A. Hinkkanen in [6] related to this problem.

1. Introduction

The systematic global study of the phase portrait of dynamical systems given by the iterates of holomorphic maps of the complex plane goes back to the work of Pierre Fatou and Gaston Julia, at the beginning of the twentieth century. They developed a theory based on the concept of normal families and described precisely what nowadays is known as the Fatou and Julia sets.

If $f : \mathbb{C} \to \mathbb{C}$ is a transcendental entire function (similar definitions apply if f is a rational function on the Riemann sphere, or if f is a transcendental meromorphic function on the complex plane), the *Julia set*, $\mathcal{J}(f)$, is defined as the set of points $z \in \mathbb{C}$ for which the family of iterates $\{f^n\}_{n\geqslant 0}$ fails to be normal in the sense of Montel in every neighborhood of z; that is, if V is a neighborhood of a point in the Julia set, the infinite union of iterates of V must cover the whole plane with the exception of, at most, two points. By definition $\mathcal{J}(f)$ is closed and it can be proven that it is infinite [12]. Its complementary domains in $\mathbb{C}$, if any, are called *Fatou domains* or *Fatou components* and the union of all those components is called the

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