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DEFORMATION OF ENTIRE FUNCTIONS WITH BAKER DOMAINS

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Abstract. We consider entire transcendental functions f with an invariant (or periodic) Baker domain U. First, we classify these domains into three types (hyperbolic, simply parabolic and doubly parabolic) according to the surface they induce when we take the quotient by the dynamics. Second, we study the space of quasiconformal deformations of an entire map with such a Baker domain by studying its Teichmüller space. More precisely, we show that the dimension of this set is infinite if the Baker domain is hyperbolic or simply parabolic, and from this we deduce that the quasiconformal deformation space of f is infinite dimensional. Finally, we prove that the function $f(z) = z + e^{-z}$, which possesses infinitely many invariant Baker domains, is rigid, i.e., any quasiconformal deformation of f is affinely conjugate to f.

1. **Introduction.** Let $f : S \to S$ be a holomorphic endomorphism of a Riemann surface S. Then f partitions S into two sets: the Fatou set $\Omega(f)$, which is the maximal open set where the iterates $f^n, n = 0, 1, \ldots$ form a normal sequence; and the Julia set $J(f) = S \setminus \Omega(f)$ which is the complement.

If $S = \widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, then f is a rational map, and every component of $\Omega(f)$ is eventually periodic by the non-wandering domains theorem in [25]. There is a classification of the periodic components of the Fatou set: such a component can either be a cycle of rotation domains or the basin of attraction of an attracting or indifferent periodic point.

If $S = \mathbb{C}$ and f does not extend to $\widehat{\mathbb{C}}$ then f is an entire transcendental mapping (i.e., infinity is an essential singularity) and there are more possibilities. For example a component of $\Omega(f)$ may be wandering, that is, it will never be iterated to a periodic component. Like for rational mappings there is a classification of the periodic components of $\Omega(f)$ (see [5]) and compared to rational mappings, entire

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