Arnold Disks and the Moduli of Herman Rings of the Complex Standard Family

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Dedicated to Bodil Branner on her 60th birthday

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Abstract

We consider the Arnold family of analytic diffeomorphisms of the circle $x \mapsto x + t + \frac{a}{2\pi}\sin(2\pi x) \mod (1)$, where $a, t \in [0, 1)$ and its complexification $f_{\lambda,a}(z) = \lambda z e^{\frac{a}{2}(z - \frac{1}{z})}$, with $\lambda = e^{2\pi i t}$ a holomorphic self map of $\mathbb{C}^*$. The parameter space contains the well known Arnold tongues $\mathcal{T}_\alpha$ for $\alpha \in [0, 1)$ being the rotation number. We are interested in the parameters that belong to the irrational tongues and in particular in those for which the map has a Herman ring. Our goal in this paper is twofold. First we are interested in studying how the modulus of this Herman ring varies in terms of the parameter a, when a tends to 0 along the curve $\mathcal{T}_\alpha$. We survey the different results that describe this variation including the complexification of part of the Arnold tongues (called *Arnold disks*) which leads to the best estimate. To work with this complex parameter values we use the concept of the *twist coordinate*, a measure of how far from symmetric the Herman rings are. Our second goal is to investigate the slice of parameter space that contains all maps in the family with twist coordinate equal to one half, proving for example that this is a plane in $\mathbb{C}^2$. We show a computer picture of this slice of parameter space and we also present some numerical algorithms that allow us to compute new drawings of non–symmetric Herman rings of various moduli.