The Teichmüller Space of an Entire Function

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Abstract

We consider the Teichmüller space of a general entire transcendental function $f : \mathbb{C} \to \mathbb{C}$ regardless of the nature of the set of singular values of f (critical values and asymptotic values). We prove that, as in the known case of periodic points and critical values, asymptotic values are also fixed points of any quasiconformal automorphism that commutes with f and which is homotopic to the identity, rel. the ideal boundary of the domain. As a consequence, the general framework of McMullen and Sullivan [McMullen & Sullivan 1998] for rational functions applies also to entire functions and we can apply it to study the Teichmüller space of f, analyzing each type of Fatou component separately. Baker domains were already considered in citefh, but wandering domains are new. We provide different examples of wandering domains, each of them adding a different quantity to the dimension of the Teichmüller space. In particular we give examples of rigid wandering domains.

1 Introduction

Given a holomorphic endomorphism $f : S \to S$ on a Riemann surface S, we consider the dynamical system generated by the iterates of f denoted by $f^n = f \circ \overset{n}{\cdots} \circ f$. There is a dynamically natural partition of the phase space S into the *Fatou set* $\mathcal{F}(f)$, where the iterates of f form a normal family, and the *Julia set*, $\mathcal{J}(f) = S - \mathcal{F}(f)$ which is the complement.

If $S = \widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, then f is a rational map. If $S = \mathbb{C}$ and f does not extend to $\widehat{\mathbb{C}}$ then f is an entire transcendental mapping, i.e., infinity is an essential singularity. Entire transcendental functions present fundamental differences with respect to rational maps.

One of them concerns the set of singularities of the inverse function. For a rational map f, all branches of the inverse function are locally well defined except on the set of *critical values*, i.e., points $v = f(c)$ where $f'(c) = 0$. If f is transcendental, there is another obstruction to inverse branches being well defined: Some inverse branches are not well defined in any

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