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On connectivity of Julia sets of transcendental meromorphic maps and weakly repelling fixed points II

by

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Abstract. Following the attracting and preperiodic cases ([FJT]), in this paper we prove the existence of weakly repelling fixed points for transcendental meromorphic maps, provided that their Fatou set contains a multiply connected parabolic basin. We use quasi-conformal surgery and virtually repelling fixed point techniques.

1. Introduction. Let f be a rational, transcendental entire or transcendental meromorphic function. We say that a p-periodic point z_0 of f is *attracting* (resp. *repelling*) if the modulus of its *multiplier* $\rho(z_0) := (f^p)'(z_0) \in \mathbb{C}$ is smaller (resp. greater) than 1, and that it is *parabolic* if $\rho(z_0) = e^{2\pi i\theta}$ with $\theta \in \mathbb{Q}$. Furthermore, z_0 is said to be *weakly repelling* if it is repelling or parabolic of multiplier 1. For any rational map of degree greater than one, the existence of at least one such fixed point is guaranteed by a theorem of Fatou [F].

As for global dynamics, points can be classified according to their long-term behaviour under iteration of the function, thus one defines the *Fatou set* $\mathcal{F}(f)$ (or simply $\mathcal{F}$ when possible) as the set of points $z_0 \in \widehat{\mathbb{C}}$ for which the family $\{f^k\}_{k\in\mathbb{N}}$ is defined and normal in a neighbourhood of z_0, and the *Julia set* as its complement, $\mathcal{J} = \mathcal{J}(f) := \widehat{\mathbb{C}} \setminus \mathcal{F}(f)$. Then a connected component of the Fatou set (Fatou component) U is called *preperiodic* if there are integers $p > q \geq 0$ such that $f^p(U) = f^q(U)$, and, more precisely, *p-periodic* when $q = 0$ and *fixed* when moreover $p = 1$. On the contrary, a Fatou component is called a *wandering domain* if it fails to be preperiodic.

According to the work of Cremer and Fatou, a p-periodic Fatou component U is necessarily one of the following: an *immediate attractive basin* if U contains an attracting p-periodic point z_0 such that $\lim_{n\to\infty} f^{np}(z) = z_0$ for

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