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DYNAMIC RAYS OF BOUNDED-TYPE TRANSCENDENTAL SELF-MAPS OF THE PUNCTURED PLANE

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Abstract. We study the escaping set of functions in the class $\mathcal{B}^*$, that is, transcendental self-maps of $\mathbb{C}^*$ for which the set of singular values is contained in a compact annulus of $\mathbb{C}^*$ that separates zero from infinity. For functions in the class $\mathcal{B}^*$, escaping points lie in their Julia set. If f is a composition of finite order transcendental self-maps of $\mathbb{C}^*$ (and hence, in the class $\mathcal{B}^*$), then we show that every escaping point of f can be connected to one of the essential singularities by a curve of points that escape uniformly. Moreover, for every sequence $e \in \{0,\infty\}^{\mathbb{N}_0}$, we show that the escaping set of f contains a Cantor bouquet of curves that accumulate to the set $\{0,\infty\}$ according to e under iteration by f.

1. **Introduction.** Complex dynamics concerns the iteration of a holomorphic function on a Riemann surface S. Given a point $z \in S$, we consider the sequence given by its iterates $f^n(z) = (f \circ \overset{n}{\cdots} \circ f)(z)$ and study the possible behaviours as n tends to infinity. We partition S into the *Fatou set*, or stable set,

$$F(f) := \left\{ z \in S \ : \ (f^n)_{n\in\mathbb{N}} \text{ is a normal family in some neighbourhood of } z \right\}$$

and the *Julia set*, or chaotic set, $J(f) := S \setminus F(f)$. If $f : S \subseteq \hat{\mathbb{C}} \to S$ is holomorphic and $\hat{\mathbb{C}} \setminus S$ consists of essential singularities, then conjugating by a Möbius transformation, we can reduce to the following three cases:

- $S = \hat{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ and f is a rational map;
- $S = \mathbb{C}$ and f is a transcendental entire function;
- $S = \mathbb{C}^* := \mathbb{C} \setminus \{0\}$ and *both* zero and infinity are essential singularities.

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