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Asymptotic size of Herman rings of the complex standard family by quantitative quasiconformal surgery

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Abstract. In this paper we consider the complexification of the Arnold standard family of circle maps given by $\widetilde{F}_{\alpha,\varepsilon}(u) = ue^{i\alpha}e^{(\varepsilon/2)(u-1/u)}$, with $\alpha = \alpha(\varepsilon)$ chosen so that $\widetilde{F}_{\alpha(\varepsilon),\varepsilon}$ restricted to the unit circle has a prefixed rotation number θ belonging to the set of Brjuno numbers. In this case, it is known that $\widetilde{F}_{\alpha(\varepsilon),\varepsilon}$ is analytically linearizable if ε is small enough and so it has a Herman ring $\widetilde{U}_\varepsilon$ around the unit circle. Using Yoccoz's estimates, one has that *the size* $\widetilde{R}_\varepsilon$ of $\widetilde{U}_\varepsilon$ (so that $\widetilde{U}_\varepsilon$ is conformally equivalent to $\{u \in \mathbb{C} : 1/\widetilde{R}_\varepsilon < |u| < \widetilde{R}_\varepsilon\}$) goes to infinity as $\varepsilon \to 0$, but one may ask for its asymptotic behavior.

We prove that $\widetilde{R}_\varepsilon = (2/\varepsilon)(R_0 + \mathcal{O}(\varepsilon \log \varepsilon))$, where R_0 is the conformal radius of the Siegel disk of the complex semistandard map $G(z) = ze^{i\omega}e^z$, where $\omega = 2\pi\theta$. In the proof we use a very explicit quasiconformal surgery construction to relate $\widetilde{F}_{\alpha(\varepsilon),\varepsilon}$ and G, and hyperbolic geometry to obtain the quantitative result.

1. *Introduction*

The *complex standard family* of self maps of $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ is given by the two-parameter family

$$\widetilde{F}_{\alpha,\varepsilon}(u) = ue^{i\alpha}e^{(\varepsilon/2)(u-1/u)},$$

where $\alpha \in [0, 2\pi)$ and $\varepsilon \in [0, 1)$. These maps are holomorphic in $\mathbb{C}^*$ and the points at 0 and infinity are essential singularities (see [**Ba, Ko1, Mak, Ke, Ko2, F**]). For small ε, these functions are perturbations of the rotation of angle α with respect to the origin. The interest in this family relies on the fact that it is the extension to the complex plane of the well-known *Arnold family* of circle maps (see [**Ar, dMvS**]). Indeed, the unit circle $\mathbf{C}_1$