

Bifurcations and Symbolic Dynamics for bimodal degree one circle maps: The Arnol'd Tongues and the Devil's Staircase

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Preface

The pourpose of the present memory is study the bifurcations and the symbolic dynamics of bimodal degree one circle maps and some related topics. The memory is organiced as follows.

In Chapter 1 we complete the work of Levi [32] in order to explain the transition, in a forced relaxation oscillator of van der Pol type, from the non-chaotic behaviour to the chaotic one. In Chapter 2 we give a characterization of the set of kneading sequences for bimodal degree one circle maps. In Chapter 3 we construct two self-similarity operators in order to study the bifurcations of continuous parametrized families of bimodal degree one circle maps. Lastly, in Chapter 4 we give a formula to compute the topological entropy of a sub-class of bimodal degree one circle maps.

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